Coulomb gas for Wishart. The Wishart ensemble of random matrices is the set of matrices W constructed by taking $X \in \mathbb{R}^{N\times M}$ with $X_{i\alpha} \sim \mathcal{N}(0, N^{-1})$ independently distributed and writing $W = XX^T$. Since the X are $N \times M$ rectangular matrices, the W are $N \times N$ square. Because of the way they are constructed, the entries of W are not independent, but the ensemble of W is an invariant ensemble because it can be written

$$\rho(W) = \frac{1}{Z} e^{-\frac{1}{2} N \operatorname{Tr} W + \frac{1}{2}(M-N-1) \operatorname{Tr} \log W} \tag{1}$$

See Chapter 13 of the Livan text for details about why this is the distribution of W. In any case, this means that the joint distribution of eigenvalues has the form

$$\rho(\boldsymbol{x}) = \frac{1}{Z} e^{-N \sum_{i=1}^{N} V(x_i) + \frac{1}{2} \sum_{i\neq j} \log |x_i - x_j|} \tag{2}$$

for $V(x) = \frac{1}{2}x - \frac{1}{2}\alpha \log x$ with $\alpha = M/N - 1$. Following chapter 5 of the Livan text, derive the spectral density of this ensemble.

Hint: Make extensive use of computer algebra, and remember to feed *Mathematica* plenty of assumptions about the positivity or reality of positive and real variables!