Population annealing on random regular graphs. We will use the population annealing algorithm to study the spectral density of ensembles $H_{ij} = C_{ij} J_{ij}$ for C the adjacency matrix of a random regular graph and J GOE with variance $\frac{1}{2}$ (not $\frac{1}{2}N^{-1}$!).

1. Write a program to generate the adjacency matrix of random regular graphs of degree p. Compare the histogram of eigenvalues of C for a large random regular graph with the spectral density derived in class for some choices of p. Now plot the histogram of H for the same p. How do they compare with the deterministic $J_{ij} = \frac{1}{\sqrt{2}}$ case?

   Hint: You can generate a random regular graph in *Mathematica* with the following one-liner:
   `randomRegularGraph[p_, n_] := RandomGraph[DegreeGraphDistribution[ConstantArray[p, n]]]`
   Then check out the `AdjacencyMatrix` command. . .

2. Implement population annealing for this ensemble of random regular graphs. Run your algorithm at $z = x + i\epsilon$ for some small regularizer ϵ and evenly-spaced x. Plot the spectral density measured by your algorithm against the histogram of eigenvalues for a particular (large) random regular graph with the same degree p.

   Hint: You will need to generate a starting population of M cavity Green functions, then each iteration replace one at random with a new cavity Green function generated by the cavity equations for p neighbors chosen at random from the population, with couplings J generated at random from their distribution. Repeat iterations until the distribution of cavity Green functions appears to have converged. Then, sample the one-site marginal Green functions from the population of cavity Green functions, and use them to calculate the average resolvent.