Random matrix theory: techniques and applications

Motivation

Originating from the study of heavy nuclei, random matrix theory now finds application in diverse disciplines, including:

- glasses and spin glasses
- quantum disordered and many-body systems
- population ecology
- economics and finance
- machine learning and statistical inference

This course will cover the set of basic techniques for analysing random matrices, including calculation of the spectral density and eigenvalue correlations. They will be applied to canonical rotationally invariant matrix ensembles, with discussion of their applications in the above fields. Finally, we will review advanced and topical extensions relevant to modern research, guided by student interest.

Program

1. The invariant ensembles of Gaussian random matrices
2. Joint probability distribution of eigenvalues & 'coulomb gas'
3. The spectral density & Wigner semicircle
4. The Wishart ensemble & Marčenko–Pastur
5. Techniques: replicas, supersymmetry, dynamics, & free probability
6. Beyond invariant ensembles: sparse matrices, low-rank perturbations
7. Applications (depending on interest): random landscapes, population ecology, finance, etc

References

[1]G. Livan, M. Novaes, and P. Vivo, *Introduction to random matrices, Theory and practice*, Vol. 26, SpringerBriefs in Mathematical Physics (Springer International Publishing, 2018).

[2]M. Potters and J.-P. Bouchaud, *A first course in random matrix theory, For physicists, engineers and data scientists* (Cambridge University Press, Nov. 2020).