Statistical mechanics of disordered systems

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Prerequisites: A course in statistical mechanics or thermodynamics.

Motivation

Disordered systems arise when part of a system is frozen while the rest evolves quickly. The interaction between frozen and fast degrees of freedom has the effect of a fixed random background on the fast part. Examples include:

System	Frozen degrees of freedom	Fast degrees of freedom
Glasses	Positions of the atoms	Vibrations and small movements
Spin glasses	Positions of magnetic impurities	Spin orientations
Neural networks	Training data	Network weights
Ecosystems	Species and their relations	Abundance of each specie
Brains	Connectivity of neurons	Activity of neurons

Techniques of statistical mechanics developed for ergodic systems must be extended to address disordered ones. We will introduce these techniques and use them to study the phenomenology of simple models. We will also preview how these methods can be applied to complex systems like those tabulated above.

Program

1. Introduction and motivation

2. Review of equilibrium statistical mechanics

    (a) The Curie–Weiss model and its 'spherical' variant
    (b) Equilibrium free energy of Curie–Weiss model
    (c) Equilibrium dynamics of (spherical) Curie–Weiss model

3. Analysis of simple models of disordered systems

    (a) The Sherrington–Kirkpatrick model and its 'spherical' variant
    (b) Annealed free energy and its failure
    (c) Quenched free energy and the replica method
    (d) Replica symmetry and replica symmetry breaking
    (e) Equilibrium dynamics: cavity method and dynamical transition
    (f) Out-of-equilibrium dynamics: aging and weak ergodicity breaking
    (g) Complexity of metastable states

4. The world of applications: neural networks, ecosystems, brains, and beyond

References

[1] M. Mezard, G. Parisi, and M. Virasoro, *Spin glass theory and beyond* (World Scientific, Nov. 1986).

[2] T. Castellani and A. Cavagna, "Spin-glass theory for pedestrians", Journal of Statistical Mechanics: Theory and Experiment **2005**, P05012 (2005).

[3] Lecture notes to be provided.